Experimental Test of the validity of "Isotropic" Approximation

for the Mechanical Behaviour of Clay

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Abstract:

Experimental data from axially symmetric compression test at constant mean pressure p=$(\sigma_1+\sigma_2+\sigma_3)$/3 *on kaolinite clay are used to study the validity of an "isotropic" modelling as a function of the overconsolidation ratio (OCR).The isotropic assumption is found to be quite good for 2<OCR<3 and/or in the range of small deformation for OCR>4. For very large OCR (OCR >10), anisotropic response is observed at few percents axial deformation. Relation with anisotropic distribution of local force is made.*

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Recent papers [1-3] have proposed a new simple modelling of the behaviour of granular media. It is based on an incremental modelling, assuming an "isotropic" response [4] governed by two plastic parameters, *i.e.* the pseudo Young modulus $1/C_o$ and pseudo Poisson coefficient ν. This modelling assumes then that the incremental response of any compression test with axial symmetry obeys at any stage of the deformation the following equation:

$$\begin{pmatrix} de_1 \\ de_2 \\ de_3 \end{pmatrix} = -C_o \begin{pmatrix} 1 & -n & -n \\ -n & 1 & -n \\ -n & -n & 1 \end{pmatrix} \begin{pmatrix} ds_1 \\ ds_2 \\ ds_3 \end{pmatrix} \qquad (1)$$

Where C_o plays the part of an inverse pseudo Young modulus and ν the part of a pseudo Poisson coefficient. σ_i states for the effective stress supported by the granular assembly alone; it does not include the pressure u_w supported directly by the liquid invading the pores. It has been proposed in [1-3] that evolution of C_o and ν has to be determined from any test of triaxial compression with axial symmetry; and it has been found from triaxial compression at constant $\sigma_3=\sigma_2$ that ν depends only the stress ratio σ_1/σ_3.

Indeed, this "isotropic" assumption may be a crude approximation even when the medium remains homogeneous with axial symmetry, since as the axial deformation

proceeds, the distribution of contacts evolves which can induce anisotropy; in turn, this anisotropy modifies the response equation, *i.e.* Eq. (1).

For instance, still assuming that the granular medium remains homogeneous with axial symmetry during the whole deformation, one expects the more general form to be valid:

$$
\begin{pmatrix} \boldsymbol{de}_1 \\ \boldsymbol{de}_2 \\ \boldsymbol{de}_3 \end{pmatrix} = -C_o \begin{pmatrix} 1 & -\boldsymbol{n}' & -\boldsymbol{n}' \\ -\boldsymbol{n} & a & -\boldsymbol{n}'' \\ -\boldsymbol{n} & -\boldsymbol{n}'' & a \end{pmatrix} \begin{pmatrix} \boldsymbol{ds}_1 \\ \boldsymbol{ds}_2 \\ \boldsymbol{ds}_3 \end{pmatrix} \tag{2}
$$

At this stage, it is worth recalling that $\nu=\nu'$ if *the* two different compression paths ($\delta\sigma_1\neq0$, $\delta\sigma_2=0$, $\delta\sigma_3=0$) and ($\delta\sigma_1=0$, $\delta\sigma_2\neq0$, $\delta\sigma_3=0$) pertain to the same incremental linear zone. It is also worth recalling that any axial compression give access to the quantity $\alpha-\nu''$ only so that α and ν'' cannot be measured separately with such axi-symmetric triaxial apparatus.

Anyhow, one can then ask whether it is useful to use Equation (2) rather than its simplified "isotropic" version, *i.e.* Eq. (1). Indeed this can be checked directly from experimental data. For instance the validity of the "isotropic" version has been discussed when applied to oedometer compression [3,5]. But this validity can be questioned in a more general way. One way to check its validity is to apply an axially symmetric triaxial compression at constant pressure $p=(\sigma_1+\sigma_2+\sigma_3)/3$, since one expects $\delta v=0$ in the case of an "isotropic" response whatever the applied deviatoric stress $q=\sigma_1-\sigma_2$.

However to investigate the variation of the specific volume v versus the deviatoric stress q and shows that v depends very little on q is not sufficient since applying Eqs. (1) or (2) allows to determine the variation law which is expected from the two approaches and to show that both contain the products $\delta\varepsilon_v=C_o\cdot f(\nu,\nu',\nu'')\cdot\delta q$ which can be small when either C_o, or f , or both are small. So it does not demonstrate that f is small. This is why one can use the same procedure as proposed in [6].

One is then faced to determine directly the function f. A better way to proceeds is just to use experimental plots of variations of ε_v as a function of ε_1 or ε_d, where. $\varepsilon_d = \varepsilon_1 - \varepsilon_2$. Since $dp=0$ imposes $\delta\sigma_1=(2/3)\,\delta q=-2\delta\sigma_2=-2\delta\sigma_3$, one gets from Eq. (2),:

$$\delta\varepsilon_v=\delta\varepsilon_d\, 2[(1-\alpha)+(\nu'+\nu''-2\nu)]/(2+\alpha+2\nu+2\nu'+2\nu'-\nu'') \tag{3}$$

When the response is isotropic, one gets $\nu=\nu'=\nu''$ and $\alpha=1$, so Eq. (3) becomes :

$$\delta\varepsilon_v=\delta\varepsilon_d\, 2[O(1-\alpha)+O(\nu'+\nu''-2\nu)]/(2+\alpha+2\nu+2\nu'-\nu'') \tag{4}$$

where O(x) is a function which tends to 0 as x. Furthermore, an estimate of α , ν, ν' & ν'' are 1, ½ , ½ & ½ respectively, since these values are those obtained for the critical state. So, $\alpha=1$ and Eq. (4) becomes

$$\delta\varepsilon_v= (4/9)[O(1-\alpha)+O(\nu'-\nu'')]\,\delta\varepsilon_d \tag{5}$$

So Eq. (5) gives the expected variations of $\delta\varepsilon_v$ *vs.* $\delta\varepsilon_d$ for an axially symmetric compression at constant mean pressure p. It indicates that the smaller the slope of ε_v *vs.* ε_d the better the "isotropic" approximation. We report on Fig. 1 variations of ε_v *vs.* ε_d under constant-pressure-p compression obtained on Kaolinite clay at different over-consolidation ratio (OCR), ranging from OCR=1 to OCR=50. These tests have been performed under the following procedure: First, each sample of clay has been compressed at $p_o=1$ M Pa; then the mean pressure p has been reduced to $p=p_o/OCR$; which is the mean pressure p which has been used during the axial compression.
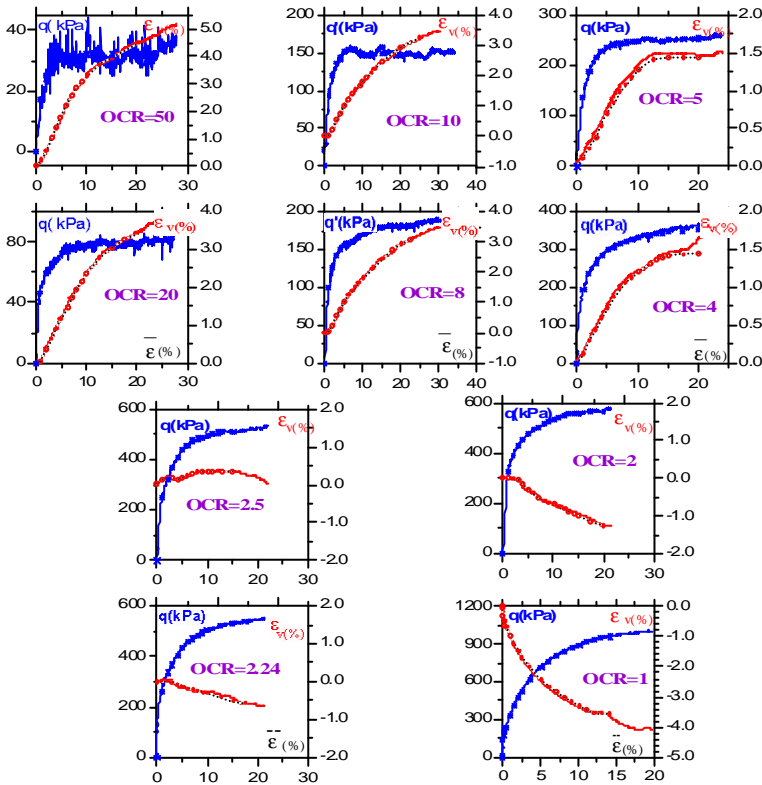
Figure 1: *variations of the deviatoric stress* q *(crosses) and of volume deformation* ε_v *(circles) vs. the deviatoric deformation* ε_d *during an axial compression at constant mean pressure* p, *for different* p *value, but on the same Kaolinite clay. All samples were compacted initially at the same over-consolidation pressure* $p_o=1$ M Pa; *each test corresponds to a different over-consolidation ratio* (OCR)*: OCR=p_o/p. OCR: 1, 2, 2.24, 2.5, 4, 5, 8, 10, 20, 50. Data from [7] .*

The ε_v *vs.* ε_d variations of Fig. 1 show that :
- an horizontal tangent at ε_d=0, except for OCR=p_o/p = 1, 4, 5 and perhaps at OCR= 50. This indicates that the response is rather "isotropic" at small ε_d and small q/p hence, if one excepts OCR=p_o/p= 1, 4, 5 & 50.

- for OCR larger than 3 one observes an increase of the slope as soon as the deformation ε_d overpasses 1%.

We report on Table 1 the maximum value of the slope $\varepsilon_v/\varepsilon_d$ for the different OCR. These data shows that the response is quite "isotropic" when OCR ranges in between 2 & 3. It is never isotropic for OCR=1. For other OCR "isotropic" approximation is good in the vicinity, *i.e.* few percent, of ε_d =0, except for OCR=4 & 5. We find also that the larger the OCR the more anisotropic the response since the larger the mean slope of ε_v *vs.* ε_d . The slope $\delta\varepsilon_v/\delta\varepsilon_d$ can be as large as 0.37 which corresponds to a slope $\delta\varepsilon_v/\delta\varepsilon_1\approx$ 0.45.

It is worth mentioning that this last point is indeed not an obvious answer, because the volume variation seems to remain small in a large range of q/p when plotting of ε_v *vs.* q or q/p. It is due to the fact that C_o decreases strongly when increasing the OCR so that the amplitude of both ε_d **and of** ε_v deformations decreases strongly with q/p at large OCR. In other terms, it means simply that E, the pseudo Young modulus, increases strongly when OCR increases.

A consequence of this is that the procedure proposed in [6] to demonstrate the existence of a bifurcation process during the undrained test, which is based on the analysis of ε_v *vs.* q curves obtained from compression tests at constant pressure p, does not assume or does not imply an isotropic response: this demonstration requires only that volume variation remains small in a range of q/p ratio laying around q/p=M', which is observed experimentally indeed.

OCR=p_o/p	1	2	2.24	2.5	4	5	8	10	20	50
Max(ε_v /ε_d)	-1 ?	-0.064	0.04	0.016	0.14	0.14	0.22	0.20	0.27	0.37
slope at ε_d =0	- ∞ ?	≈0	≈0	≈0	?	?	≈0	≈0	≈0	?

Table 1: maximum slope of ε_v vs. ε_d and slope of ε_v vs. ε_d at the origin of deformation for the different axially symmetric compression test at constant pressure p, for the same kaolinite clay for the same over-consolidation pressure p_o=1 M Pa, but at different over-consolidation ratio (OCR=p_o/p); *data from* [7].

This demonstrates that generation of anisotropy does not require the evolution of contacts, in clays at least, since anisotropic behaviours appear at small ε_d already. This may be the experimental proof of the existence of a process able to generate a pure stress-induced anisotropy, without the requirement of strain generation. This result may then reinforce the results from numerical simulations on granular media obtained by Radjai and coworkers [8] who have observed two force networks, one which remains isotropic and corresponds to an isotropic answer, and the other which is the answer of the granular medium to an anisotropic stress [8] and which generates an anisotropic response. This is important to remark since some previous papers of one of us [2,3,5] was assuming that development of anisotropic answer was requiring the generation of an important axial strain (ε_1>5%). On the contrary, the experimental data on clays seems to infirm this hypothesis.

At last, it is worth noting a final point, which concerns the q *vs.* ε_d curves: Fig. 1 data demonstrates that q tends to a limit value such that q/p≈1.5 – 1.6 at large deformation and for large OCR (OCR>5). Furthermore, the larger the OCR the sooner, *i.e.* the smaller ε_d value at which, this q/p=1.5 limit value is reached. No doubt, these facts shall be attributed to the divergence of C_o , (or E→0). So considering Eq. (2) and combining it with the experimental condition δp=0 impose that $\delta\varepsilon_d=[C_o/3]\delta q(2+\alpha+2\nu+2\nu'-\nu'')$ and $\delta\varepsilon_v=[C_o/3]\delta q(2-2\alpha-4\nu+2\nu'+2\nu'')$. In other words, the increase of Co imposes the increase of the volume variation; this is why one observes a large volume variation at large q/p in these large-OCR cases. It is also worth noting that this q/p ratio is larger than the expected one from natural friction of clay, since M'=1.5 or 1.6 leads to 37°< φ <39° , according to sinφ=3M'/(6+M'). This typical value of M' which is larger than the typical one of the critical state is probably induced by the dilatancy mechanism which is still visible from the experimental data, so that one should expect a decrease of the q/p ratio at deformation ε_d larger than 25%.

As a conclusion this paper shows from experimental data on axially symmetric compression of Kaolinite clay at constant pressure that "isotropic" response is valid (i) in the whole domain of deformation for 2<OCR<4 , (ii) in a small range of axial deformation near ε_d=0 for OCR >4.

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[4] When "isotropic" is written in bracket, it means that the matrix which represents the incremental stress-strain law is isotropic. It does not mean that the real material behaviour is isotropic, since the material complete response is non linear: Assuming that the material behaves isotropically would mean that its total response function is determined by the deviatoric stress q and the mean pressure p' though a function f(q,p') which is non linear. From derivation of f against q and p', one finds the total deformation tensor and second derivation leads to the determination of the incremental response.

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