Discussion on paper P&G 7, 1-18 (1999),

i.e. about test of "BCCW theory"

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Abstract :
Testimony #1 was produced to "la Cour administrative d'Appel" in Paris; so the following correspondence is no more private but open to anybody and can be used by anybody refereeing to it.

Pacs # : 5.40 ; 45.70 ; 62.20 ; 83.70.Fn

This experiment (reported in [1] was first settled after few discussion with JP. Bouchaud on the BCCW theory [2]. Its results surprised the authors of BCCW theory.

I presented this experiments at Powders & Grains meeting (1997) and at the 1-year session on granular matter at KITP Santa Barbara…

In a first time, I proposed to publish these results with J.P. Bouchaud who did not accept. So I send the paper [1] to J. de Physique for publication. After few negative reports, I decided to transform Poudres & Grains in its new form (*a posteriori* peer reviewing, to see what happens.

See also the Cates report on this paper.
No comment on this paper was received by the Journal for publication. I did not ask M.Cates if I wanted that I print his report. Nor he asked for…
Perhaps this should have been asked by the administration or from editors; but I did not received such a claim. So I considered I could not print the reports… till I was allowed but sending my testimony the Court in 2016. I tried to discuss the problem with the administration, … No answer, till I was put in "longue maladie d'office for no reason and without my agreement".

This is the proof that Administration thinks it is right what ever it does. It is not acceptable in Science. It breaks safety rules of working in labs; it breaks also safety rules based on scientific management of equipment (Nuclear Plants, …)

References:

[1] P. Evesque: Stress propagation in granular media: Breaking of any constitutive state equation relating local stresses together by a change of boundary conditions, P&G **7**, 1-18 (1999),
[2] J. P. Bouchaud, M.E. Cates & P. Claudin, J. Phys. I France **5**, 639, (1995); and further refs in [1]
[3] http://defense-pierre-evesque.over-blog.com/ in general; and http://www.poudres-et-grains.eu/datas/suite_affaire_2/3rr-mem-22.4.16-CAA.pdf, for making public the private peer-reviewing correspondence.
[4] http://poudres-et-grains.eu/datas/temoignages/Temoig-1_editionsCL-23-6-11.pdf , pp. 218-230

Problème du modèle BCCW, rejet d'article surla théorie BCCW :
cf. Poudres & Grains n°7, 1-18, (1999)

J'ai déjà relaté dans ma notice de Titres et Travaux 2001 et dans mon rapport cnrs 2011 le rejet de cet article par le Journal de Physique.
L'expérience que je propose a été montée avec l'accord de JP Bouchaud. Cette expérience a été exposée au congrès de Powders & Grains 2007, puis au KITP de Santa Barbara, USA, pendant un mois, sans réellement qu'elle soit discutée par les physiciens présents.
Je ne pense pas que les critiques des référees soient correctes

Il est malheureux qu'aucune interprétation théorique pour expliquer le résultat n'ai été réellement propposée par les contradicteurs de cet article, voir aussi le rapport de M. Cates.

cf: Titres et Travaux de P.Evesque 2001 (puis suivant) et dans rapport cnrs 2011 de P.Evesque

Témoignage de P. Evesque *23/6/2011* 12

P. Evesque, Testimony #1, CL MSSMat on 23 June 2011 p.73/238

ANNEXE 9 du *Rapport d'Activité de P. Evesque, CNRS 2009-2010 p.*57/80, reported from P. Evesque, Testimony #1, at CL MSSMat on 23 June 2011 p.219 to 230, published through **Poudres & Grains**
(And transmitted thoough Titres et travaux de P. Evesque - année 2000-2001 ; ANNEXE II)

Rapport de peut-être M. Cates sur un article cherchant à critiquer son point de vue et version corrigée de cet article incluant les réponses souhaitées.

First referee: Referee's report on: Experimental Test of Stress Propagation... by P. Evesque.
(This report is not anonymous. The author is M. E. Cates.)

The models of [1-3] work in the following way: a granular assembly is built which is assumed to have some "engraved" texture, and an incremental load applied. This propagates by rules determined from the texture, leading, within the simplest models, to a two-peaked localized response. As a result, a load applied to the top of the assembly leads to twin localized forces at the base (assumed to be rough and rigid). An obvious problem with the approach is to decide what happens if one chooses to apply a different force distribution from the one calculated. This is discussed in our recent papers (*), which conclude that this will necessarily lead to some reconstruction of the texture. This sensitivity, called "fragile" behaviour, is closely related to what Evesque means by "breaking of any constitutive state equation....by a... change of boundary conditions".

Turning to the present paper, it is in two parts; the first describes an experiment that was aimed at testing the ideas of Refs.[1-3] and the second is a general discussion based on previously published experimental results.

The experiment did not succeed. A very high compliance transducer (soft spring) is used. But most experimenters in granular matter insist absolutely on using the lowest compliance transducers possible. A soft transducer requires that any incremental change in stress is accompanied by a large displacement, and it is very well known that this will lead strong local reconstructions and therefore completely alter the state of stress that is supposedly being "measured". In this case, the soft transducer completely prevents one from measuring any large, localized stress responses that might arise in accord with Refs.[1-3].

On the other hand, the same models predicts that, whenever the transducer does not lie on one of the two "rays" of force, no reconstruction is needed and negligible force will be measured. This is is exactly what Evesque observes, although it is not at all what one would normally expect for an elastic pile on a properly rigid support. So the data could equally be taken as evidence in favor of [1-3], as against it! But in reality, because the spring is soft, it will always register negligible force, and this experiment is quite useless at distinguishing one type of stress propagation from another. Evesque's remark (p.5) that "no precise conclusion can be drawn" is somewhat understated. No conclusion whatever can be drawn by performing such an experiment.

Turning to the second part, the author appeals to "well-known classical results" on the mechanical behaviour of granular material. The main point that the author makes is that in such tests the material can be clearly observed to reorganized under the imposed stresses: the possibility of "engravement" of stress propagation rules via the material texture is then denied. He is quite correct to point this out. However he forgets that in these "classical experiments" the stresses are quite enormous compared to those arising in a freestanding bed of sand under gravity, which was the context for which the models of Refs.[1-3] were developed.

Evesque is entitled to be critical of [1-3] for not discussing clearly the range of stresses under which such models might apply. (More recent work (*) shows how to establish a crossover with conventional elastoplastic behaviour at much higher stresses.) But in fact he does not make this justified criticism. Instead he presents the case as though these triaxial experiments show the models of [1-3] must be invalid even in their original context. This is not justified criticism: experiments involving the application of enormous stresses, which cause obvious reconstructions of the texture, cannot provide any useful information concerning the much more delicate issues of stress propagation in sand under gravity.

On the positive side, the paper does make the two following valid and important points:

(1) Although the experiment entirely fails to test the nature of stress propagation in granular media, there is, nonetheless, a valid sense in which it shows "breaking of any constitutive state equation...by an adequate change of boundary conditions". It does this, in effect, by using a test in which the incremental stress measured at the transducer is automatically required to be negligible (due to the soft spring). This is clearly a case where the experimenter chooses (consciously or otherwise) to impose a given value of the stress at the boundary, rather than being able to measure such a stress. This case is fully covered by the "fragility" arguments in our recent articles (*).

(2) The models of [1-3], whether or not they are valid for poured sand under gravity as proposed (which remains to be fully tested) could easily break down under the completely different conditions, involving much larger stresses, that hold in triaxial soil mechanics tests. But conversely, such tests could well reveal nothing about sand under gravity.

As an author of [1-3], I do not wish to deny publication of rival viewpoints. Indeed, I certainly could not object to an article by Evesque in which these two valid points are clearly spelled out. Actually, Evesque's ideas on the first point have already strongly influenced our more recent work (*).

However, in the present article these points are not clearly made and remain hidden among a number of scientifically invalid, obscure or irrelevant discussions. In view of these defects, I obviously cannot recommend publication of this article in its present form.

References:
(*) J. P. Bouchaud et al, Models of stress propagation in granular media, in: Physics of Dry Granular Media, eds H. Herrmann, J-P. Hovi and S. Luding, pp 97-121, Nato ASI series E vol 350, Kluwer 1998; M. E. Cates et al., condmat 9803197 and cond-mat 9803266, to appear in Phys. Rev. Lett. and Phil. Trans. Roy. Soc. Lond., respectively, 1998.

Annexe 9, p.2

Note de PE : La qualité de l'article cité, dont je joins la version corrigée, ne doit pas être évaluée pour sa valeur intrinsèque (qui est faible car il ne contient aucun élément nouveau pour un mécanicien) mais parce qu'il tente de répondre à un certain nombre de questions que se pose la communauté des physiciens à l'heure actuelle en se basant sur des résultats expérimentaux.

The proposed Article (see also P&G 7**, 1-18** (1999)

Experimental test of stress propagation in granular media:

Breaking of any constitutive state equation relating local stresses together by an adequate change of boundary conditions

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Abstract:
Stress response of a granular assembly subject to different changes of boundary conditions is studied experimentally in order to define the stress propagation characteristics and to study the existence of a constitutive law between stresses. When stress propagation experiment is performed in a 2d pile by applying a local stress somewhere and by measuring the generated perturbation, it is demonstrated that boundary conditions may perturb strongly the expected result without imposing the need of a break down of the constitutive relation so that no conclusion can be drawn in many cases due to the lack of information. In a second part, classical results of granular-material mechanics are examined, for which boundary conditions are under complete control; these results demonstrate that no simple and single relationship between local stresses exists in general, which would be imposed by the local structure of the granular assembly only. On the countrary, it is demonstrated that these results are controlled by the boundary conditions themselves and that a tiny change of them may lead to strong variations in the incremememtal-stress relation; furthermore, in other cases, these changes may generate large variations of the stress field and can allow to understand partly the fluctuations already observed in these media.

Short title: *Experimental test of stress propagation*
PACS: 46.10+z ; 81.05.Rm ; 83.70Fn

Introduction

The mechanics of granular media have been the subject of many studies in recent years [1, 7]. In particular, large interest has been carried on the prediction of the stress distribution in these materials and on local stress fluctuations [1-3]. All started by a simple remark of a group (Bouchaud, Claudin , Cates, Wittmer, refered as BCCW in the following text), which says that if stress tensor $\underline{\sigma}$ can be defined easily in a granular material because the intergranular forces are well defined quantities [8], this is not the case for the strain tensor, since the material is made of grains in contact, that these grains move and slide randomly on top of one another when deformation proceeds and that the contact distribution evolves permanently; in other words, deformation generates so many discontinuities that it makes null the assumption of continuity which is needed to define strain.

Annexe 9, p.3

Thus, it has been considered by three of these authors (BCC) that the old mechanical approach [9,10] based on an elasto-plastic mechanical modelling (and hence on strain) was not valid for these materials [11] and they tried a new mechanics approach based on stress only. As mechanical equilibrium is preserved, it implies that the stress tensor $\underline{\sigma}$ is symmetric ($\sigma_{ij}=\sigma_{ji}$) and obeys the set of 2 (or 3) differential equations $div\underline{\sigma}=\rho g$ in 2d (or 3d), where ρ is the density of the material and g the gravity. However, this is not enough to close the mathematical problem since $\underline{\sigma}$ has 3 (or 6) independent components in 2d (3d) ; thus, BCC have assumed also that 1 (3) closure relation(s) between stress components shall exist to close the problem ; they have assumed that these closure relation(s) are engraved during building of the pile and are linked to the real contact network ; this is why they called it (them) constitutive relation(s). As an example of the existence of such a closure relation, they have given the example of a silo for which it is often assumed that σ_{xx} is proportionnal to σ_{zz} (*i.e.* $\sigma_{xx}=k_2\sigma_{zz}$) [12] .

Applying their model to a 2d deep horizontal layer of rods and writing the closure equation in the form $\sigma_{xx}=tg^2(\theta)\sigma_{zz}$ (*i.e.* $k_2=tg^2(\theta)=c_o^2$), they got [1] that stress obeys the differential equation $!^2\sigma_u/x^2 - tg^2(\theta)^2\sigma_u/z^2=0$ (with u standing for xx, xz or zz), so that a stress perturbation shall propagate along two lines inclined at the angle θ compared to the vertical. So, if one applies some force $\delta F=s\delta\sigma_{zz}$ localized on a small portion s located at x_0 of the top free surface of a horizontal layer of depth h, this stress shall propagates linearly downward in the two directions defined previously and shall generate two local responses $\delta F_\pm=\delta F/2$ at the two locations $x_\pm=x_0\pm h\,tg(\theta)$ of the bottom surface. Different attempts to demonstrate this effect have been performed without success, but their results have not been published [13].

On the other hand, in a first unpublished version of the present paper, the BCC approach has been discussed in view of different experimental results and of few theoretical considerations . Among them, emphasis was given on the important part plaid i) by strain and ii) by boundary conditions [14]. In particular, classical experimental results on stress-strain behaviour of granular media obtained with triaxial apparatus was recalled briefly in order to show the role plaid by strain in general and in the case of a silo in particular [14]; these triaxial results have been used also to prove that an adequate change of boundary conditions breaks the constitutive relation. *Recent rumour however seems to claim that the BCC approach is not concerned by the mechanics observed with triaxial tests since this one applies too large stresses [15]; one shall not accept this remark and this objection will be discussed carefully in the last part of this paper.*

Besides, this previous version was reporting also some results of a 2d experiment which has been built under the advices of Bouchaud and Claudin to test the propagation of stress in a 2d horizontal deep layer of rods; this experiment has been measuring the change of stress $\delta\sigma(x_1,z=0)$ at a single point $(x_1,z=0)$ of the bottom layer when a small vertical increment of force $\delta\sigma_0(x,z=h)$ has been going over the top free surface. According to the BCC theory of stress propagation which has been recalled above, one would expect to observe a non zero increase of stress $\delta\sigma(x_1,z=0)$ when the small increment of stress $\delta\sigma_0$

is applied in ($x_{\pm}= x_1 \pm h tg(\theta)$, $z=h$). As it will be indicated below, this experiment has failed to find the expected result, since no increase of stress has been found. However, there is a good reason for that, since the experiment has been biased and since it has been using a very soft spring (*i.e.* much softer than the base supporting the pile) to measure the local stress so that the experiment has been working at constant imposed stress at (x_1, $z=0$)! Nevertheless, this experiment led to conclude (wrongly, as it will be shown later) that the closure equation was broken by any change of boundary conditions. And this result has got a strong impact in the literature [16, 17]. For instance, the BCC approach has been modified into the BCCW theory [3], in order to incorporate the notion of fragile matter and to predict the existence of large fluctuations [4]. The fragile matter concept states that the closure equation is "engraved" in the material, but that it breaks down as soon as any even infinitesimally tiny change of the contact distribution occurs; granular matter has then been proposed to be the archetype of this fragile matter and the closure equation has been called also a constitutive equation.

However, *the conclusion of the 2d experiment is questionable* indeed as it has been asserted earlier; the reason can be stated as follows: according to the BCC approach, stress $\underline{\sigma}$ is the solution of a set of partial differential equations which includes the closure equation; so, as any solution of differential equation, this solution does depend on the boundary conditions and is determined by them. Thus if one wants to prove that the closure relation is not satisfied in the present case, one can proceed in two different ways: i) either to measure $\underline{\sigma}$ somewhere and to show that it does not obey the closure equation or ii) to demonstrate that there is no solution of the set of differential equations which obeys the peculiar boundary conditions. In the present case, none of these two conditions are satisfied since it exists a

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solution of stress $\underline{\sigma}$ which is compatible with the experimental boundary conditions and with the experimental data; this will be demonstrated in section 2.

This looks trivial, but I got just a small comment only about this point. This argument turns out to be important, since it may work also elsewhere; for instance in the case of photoelastic measurement for which one finds i) that stress condenses along local pathes and ii) that stress pattern can change abruptly for a tiny increase of stress, the fact that a stress path jumps suddenly from one location to another one does not imply the breaking of the average macroscopic constitutive law as it seems to be assumed implicitly in few recent papers [3, 4, 18].

So, owing to the large impact these 2d experiments have got in the recent physics literature [16,17], it is worth recalling our own data (which are still unpublished) and it is worth discussing any dubious interpretation they lead to. It is also worth recalling few old results obtained with triaxial or biaxial set up to show how stress-strain behaviour of granular matter can be applied to demonstrate the breaking of any closure equation by an adequate change of boundary conditions; however, it will be demonstrated that the break down of the constitutive equation when it occurs is smooth, which means that the break down concerns the small increment of stress and not the complete stress so that constitutive relation cannot be tested by measuring the law of propagation of an increment of stress.

The paper is built as follows: the first part recall the 2d experiment, its goals and some of the main experimental results; the second part shows that these results cannot be used to infirmed the existence of a stable constitutive relation between the stresses. In the third part, triaxial test results are briefly summed up to demonstrate that no constitutive relation can survive to an adequate change of boundary conditions; it will be demonstrated that the break down of the constitutive law concern the incremental part $\delta\underline{\sigma}$ of the stress only, *i.e.* this one whose propagation mode should obey the closure relation, the main part $\underline{\sigma}$ of the stress tensor still continuing to obey to the closure relation at zeroth order in $\delta\underline{\sigma}$. It will be demonstrated that these results are in agreement with recent meanfield approach proposed in [18]. The end of this third part will be devoted to demonstrate that the triaxial test results can be applied to granular media submitted to small stresses since some doubt seems to appear about this point [15].

1- 2D experiments of local loading:

•1-a 2D experimental set-up:

The set-up is sketched on Fig.1. The rectangular pile is a 2d horizontal layer made of parallel duralumin cylinders of 5mm diameter, 6cm long and m_o=3.1g mass, with their axes parallel to one another and horizontal; its internal structure is dense (*i.e.* triangular lattice). This pile is confined laterally between two vertical walls whose normal vectors are parallel to each other and perpendicular to the rod axes. The pile repose also on two horizontal parallel girders fixed rigidly to the vertical walls, so that the walls and the girders form an unique rigid structure. The ends of the rods of the lower row of the pile repose on the girders, an end on a girder, the other one on the other girder. The distance between the two girders is 3.5cm and between the two vertical walls is 16 cm; both walls lay on a spring balance, labeled S_1 and S_3.

Local stress can be applied on the top free surface of the pile by loading some weight at a precise location of this surface. The way stress has been measured at any given location of the bottom of the pile is as follows: if rods of the bulk are cut by half in the direction of their length they can be inserted in the bottom row of the pile instead of initial rods without disturbing the packing and in such a way as they do not touch the girders (since they are 3cm long and since the distance between the girders is 3.5cm); obviously, they need to be held up in order not to fall down, so that these half rods can be used as vertical-stress probes if one connect them to a third spring balance S_2 (see Fig. 1) using a small device which passes in between the two girders without touching them. The balance S_2 itself can move up and down using a special carriage in order to adjust its height to maintain the probe in contact with the pile. Gluing few half rods together allows to make probes with different test area so that one can measure the width of any localized response. During the probe insertion, much care has been taken to avoid any perturbation of the internal contact structure of the pile. (And we have checked carefully this point).

Three different spring balances have been needed, two of them are hard, the other one much softer, its stiffness K=7.33µm/g =0.733mm/N. Thus, experiments have been repeated after permutation of the balances in order to check the effect of the spring rigidity. Furthermore, piles with different heights varying from H_m=3cm (*i.e.* 7 layers) to H_M=11.3cm (26 layers) have been studied and they all lead to similar results, which are not those predicted by ref. [1].

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•1-b 2d experimental results:

The experiment consists in measuring the loads S_{1o}, S_{2o} and S_{3o} carried by each balance and their variations $\delta S_{1o}(F(x))$, $\delta S_{2o}(F(x))$ and $\delta S_{3o}(F(x))$ when an extra-force F is applied at some position x on top of the pile. If the pile obeys the closure equation assumed in BCC, one shall observe an increase of $\delta S_{2o}(F(x))$ if F is localized at a position x which is related to the position x_2 of the second balance S_{2o} , according to $x=x_2 \pm h\ tg(\theta)$ as mentioned in the introduction.

However, it turns out that experimental results do not follow this prediction; but they can be summarized as follows:

(a) if F is kept equal to 0, S_{2o} can be adjusted within a given range ($0<S_{2o}<S_{2max}$) by varying the vertical position of the middle balance S_2 ; it has been found also that the sum $S_{1o}+S_{2o}+S_{3o}$ is constant for a given pile and depends linearly on the number of layers of rods forming the pile; it verifies the relation $S_{1o}+S_{2o}+S_{3o}$ =Mg , where Mg is the weight of the pile and of the structure. The contact network does not deform when changing S_{2o}.

(b) applying now some extra-force F at some position x on the top, one measures the variations $\delta S_{1o}(F(x))$, $\delta S_{2o}(F(x))$ and $\delta S_{3o}(F(x))$; these variations are reported in Fig. 2 as a function of x for different values of F and different positions of the "soft" scales. So, it is found:

(bi) These results do not depend on the size of the probe.

(bii) variations $\delta S_{1o}(F(x))$, $\delta S_{2o}(F(x))$ and $\delta S_{3o}(F(x))$ depend linearly on x and on F

(biii) this linear dependence of $\delta S_{io}(F(x))$ depends itself on the position of the "soft" scales

(biv) each set of data satisfies the relation $\delta S_{1o}(F(x)) + \delta S_{2o}(F(x)) + \delta S_{3o}(F(x))$ =F.

(bv) variations of $\delta S_{1o}(F(x))$, $\delta S_{2o}(F(x))$ and $\delta S_{3o}(F(x))$ do not depend on the pile height h.

(bvi) data satisfy never the relationship $\delta S_{2o}(F(x))$ =0 or F/2 depending on the position x of the applied extra-force F, as it is predicted by the theory of ref [1] .

(c) Furthermore, same experiments have been repeated on piles with strongly disordered structure and

(c) Furthermore, same experiments have been repeated on piles with strongly disordered structure and lead to the same results; this means that the local internal structure of the pile has little effect. Furthermore, no evolution of the contacts of the packing structure has ever been observed during loading or unloading.

(d) No variation of the contact network between the rods seems to be observed when load is applied, as far as the load is small enough and the pile is dense enough; this is true for both the triangular lattice case and for the disordered ones.

Obviously, these results do not seem to agree with model of ref [1] . But prior to discuss this with some extend, it is shown first that these results can be interpreted with classical concepts of mechanics.

•1-c Interpretation of 2D experiments:

A simple way to understand these results is to consider the pile as rigid; this is indeed a good approximation as far as the force S_2 is not too large so that the grains and the contacts do not move (*cf.* point d). Consider the pile and the structure which carries it as a whole; this system is submitted to a force F located in x and to three forces S_1, S_2 and S_3 applied by the scales in $x_1=0$, x_2 and $x_3=L$. Furthermore, as one of the balance is soft and the two others are rigid, the force applied by the soft scales remain constant about. So, equilibria of forces and of angular momenta imply:

$$F+Mg=S_1+S_2+S_3 \qquad (1a)$$

$$Fx+Mgx_G=S_2x_2+S_3L \qquad (1b)$$

where x_G is the absicsse of the center of gravity. Result (a) is explained by Eq. (1a) for which F=0 indeed. Furthermore, when F is not null, Eq. (1) leads to a unique solution if one knows x, F, M, x_G, x_2 and one of the force S_i; as the force applied by the soft scales (either S_1, S_2 or S_3) remains constant before and after the loading of F, one gets from Eq. (1):

if S_2 is the soft scales

$$\{ S_2=cste , \quad S_1=[F(L-x)+Mg(L-x_G)-S_2(L-x_2)]/L , \quad S_3=(Fx+Mgx_G-S_2x_2)/L ,$$
$$\Delta S_1=F(L-x)/L , \quad \Delta S_1=F(L-x)/L , \quad \Delta S_2=0 , \Delta S_3=Fx/L \} \qquad (2a)$$

and if S_3 is the soft scales

$$\{ S_3=cste\ ,\quad S_1=[F(x_2-x)+Mg(x_2-x_G)+S_3(L-x_2)]/x_2\ ,\quad S_2=(Fx+Mgx_G-S_3L)/x_2\ ,\quad \Delta S_1=F(x_2-x)/x_2\ ,\ \ \Delta S_2=Fx/x_2\ ,\quad \Delta S_3=0 \} \qquad (2b)$$

Indeed, these behaviours are observed respectively in Figs. 2a and 2b and explain behaviours (bi), (bii), (biii), (biv), (bv), (bvi), (c) and (d). It means that this approach looks correct.

2- *Consequences about stress propagation in the pile :*

•2-a can one conclude that stress does not propagate along lines with this experiment?

Let us first take the point of view of the BCC theory which states that stress propagates along lines. When S_2 is the soft balance, everything seems to occur as if point S_2 was repealing the stress. On the contrary, when S_2 is a stiff balance, S_2 attracts always partly the added stress and the attracted part depends on x. So, the experimental results reported here seems to deny the validity that stress propagates along straight lines: in the present case, the stress may be deviated (attracted or repealed) by some points when some special boundary conditions are imposed. This was the conclusion reported in the first version of this paper; it has been agreed by BCCW in refs [3, 4] so that these authors have modified the BCC theory in order to incorporate the possibility that the closure relation be quite sensitive to any change of boundary conditions. Hence, BCCW have solved this problem by introducing the notion of fragile matter which states that any small change of the contact network breaks down the constitutive relation and they have related the observed results (a-d) to a sensitivity of the contact network to some change of boundary condition [19].

However, as mentioned in the introduction, the whole argumentation turns out to be wrong since it is not complete for the following reason: since BCC theory [1] respects the basic requirements of equilibrium (zero force and zero torque), it certainly cannot lead to results in contradiction with Eq. (1) and hence with the experimental data. In other words, BCC theory imposes a set of differential equations which includes the constitutive equation, the solution of which depends on the boundary conditions. *So, BCC approach does not impose the boundary conditions and the stress field it predicts shall depend on boundaries*. Nevertheless, it can occur that some set of differential equations have no solutions obeying some set of boundary conditions. So, if one wants to demonstrate that BCC theory fails to explain these experimental results, one is faced to demonstrate that the set of differential equations has no solution which obeys the experimental boundary conditions (which include the measured and imposed local forces). This is just what is done in the next paragraphs.

The set of differential equation to be solved is [20]

$$\{\mathrm{div}\underline{\sigma}=\rho\underline{g}\ ,\ \sigma_{xx}=c_o^2\,\sigma_{zz}\ \} \qquad (3a)$$

with $c_o=(k_2)^{1/2}=\mathrm{tg}\theta$ and with the following boundary conditions:
At the top free surface $\sigma_{zz}(x,z=0)=0$ everywhere except in $(x_o, z=0)$ for which $\sigma_{zz}(x_o,z=0)=\sigma_o$; so, $\sigma_{zz}(x,z=0)=\sigma_o\delta(u-x_o)$.
At the bottom, boundary condition $\sigma_{zz}(x,z=h)$ is unknown except in $(x_1, z=0)$ for which $\sigma_{zz}(x_1,z=0)=\sigma_1$.

It is known from ref. [1] that solutions of this problem are of the kind:

$$\sigma_{zz}(x,z)=\sigma_+(x-c_oz)+\sigma_-(x+c_oz)+\rho gz \qquad (3b)$$

with σ_+ and σ_- are two functions to be determined using boundary conditions. For sake of simplicity, it will be assumed that the pile has an infinite length. Two cases have to be examined, the first one concerns the pile with no load on top and supported by the three balances, the second one concerns the pile with some load on top and crried by the three bances.

** Case a: pile with no load on top:*

So, for sake of simplicity, we begin describing point a of experimental results. In this case, the unloaded top boundary implies $\sigma_{zz}(x,z=0)=0$ whatever x. This condition allows to write $\sigma_+(u)+\sigma_-(u)=0$, and to replace one of these functions by the other one in Eq. (3b); so, one gets:

$$\sigma_{zz}(x,z)=\sigma_+(x-c_oz)-\sigma_+(x+c_oz)+\rho gz \qquad (4a)$$

and hence:

$$\sigma_{zz}(x,h)=\sigma_+(x-c_oh)-\sigma_+(x+c_oh)+\rho gh \qquad (4b)$$

which has to be compatible with results a, which tells that the stress at a precise location can vary at will. Indeed, this is possible since $\sigma_+(u)$ can be written as the infinite sum of the stress field at different points of the bottom.

$$\sigma_+(x-c_oh) = \sum_{p=0} \{\sigma_{zz}(x+2pc_oh,h)-\rho gh-x_o)\} = \sum_{p=1} \{\rho gh -\sigma_{zz}(x-2pc_oh,h)\} \qquad (4c)$$

This means that $\sigma_+(u)$ has a solution which depends on the stress at a discrete infinite number of points of the bottom. So, the set of differential equation has a solution under the peculiar boundary conditions of the experiment and theory [1] is able to describe the results of the present experiment which measures one component of stres at the bottom only. For instance, let us assume that $\sigma_{zz}(x,h)$ is equal to ρgh everywhere except in a given location x_o where $\sigma_o=0$ is imposed; in this case $\sigma_+(u)$ is the sum of two half cumbs of Dirac function of spatial period λ (with $\lambda=2c_oh$) along x, one has a negative amplitude, starts at x_o+c_oh and runs towards positive x and the other one has a positive amplitude, starts at x_o-c_oh and extends towards negative x. So, this theory [1] is compatible with result a.

** Case b: pile with some load on top located at x_o:*

If the boundary conditions were equivalent to the infinite medium, results given in [1] would be expected (*i.e.* two increases of stress at the two locations x_o-c_oh and x_o+c_oh of the bottom). However, this is not true here where the experiment works with constant stress at some points; in this case, the expected increase of stress at the bottom location where constant stress is imposed shall be compensated by a local decrease of the initial stress distribution. As solutions of a set of differential equations form a vectorial space, one can use the additivity property and finds the new stress field by applying Eq. (4) to get the new stress field satisfying the boundary condition at these points and to determine the new function $\sigma_+(u)$.

Anyway, one can proceed directly from the general boundary conditions and Eq. (3) to get the adequate solution: Eq. (3b) with the adequate following boundary conditions , *i.e.* $\sigma_{zz}(x,z=0)= \sigma_o\delta(u-x_o)$, for the upper surface leads to

$$\sigma_+(u)+\sigma_-(u)=\sigma_o\delta(u-x_o) \quad \text{or} \quad \sigma_-(u)=\sigma_o\delta(u-x_o) - \sigma_+(u) \qquad (5a)$$

so that the stress $\sigma_{zz}(x,h)=\rho gh+\sigma_-(x-c_oh)+\sigma_-(x+c_oh)$ depends on a unique function $\sigma_+(u)$:

$$\sigma_{zz}(x,h) = \rho gh + \sigma_o\delta(x+c_oh-x_o) \ +\sigma_+(x-c_oh)-\sigma_+(x+c_oh) \qquad (5b)$$

as previously, $\sigma_+(u)$ shall be determined by the bottom boundary condition; it has always a solution whatever the distribution of $\sigma_{zz}(x,h)$ is; this one can be written as an infinite series:

$$\sigma_+(x-c_oh) = \sum_{p=0} \{\sigma_{zz}(x+2pc_oh,h)-\rho gh- \sigma_o\delta(x+[2p+1]c_oh -x_o)\}$$

$$\text{or} \qquad \sigma_+(x-c_oh) = \sum_{p=1} \{\rho gh+ \sigma_o\delta(x-[2p-1]c_oh -x_o)-\sigma_{zz}(x-2pc_oh,h)\} \qquad (5c)$$

So, this demonstrates that $\sigma_+(u)$ has a solution which depends on the stress at a infinite number of points of the bottom. So, the set of differential equation has a solution under the peculiar boundary conditions of the experiment and theory [1] remains able to describe the results a, b, c, d of the present experiment. Hence, BCC model is not so sensitive to change of boundary conditions; this does not mean in counter part that it is a good model for granular material.

Furthermore, one can even imagine some experiment which could be performed on an elastic material, for which relation $\sigma_{xx}=k_2\sigma_{zz}$ is not satisfied, but which leads to two localized increase of the stress at two points of the bottom boundary when a load is added on a single point of the top free surface, so that this experiment mimics results expected from [1] without the material obeys any closure equation. The sketch of such an experimental set-up is given in Fig. 3.

Thus, is there any stress relation similar to $\sigma_{xx}=k_2\sigma_{zz}$ really engraved in the pile structure ? This is still a question under debate at this point; nevertheless, next section shows how one can conclude no in view of well-known classical results on the mechanical behaviour of granular material [22-24].

3- 3D experiments on granular media:

In ref. [1], it is assumed the existence of some closure relationship between the stresses; an example of such a relation is $\sigma_{XX}=k_2\sigma_{ZZ}$. It is also assumed that this relation depends on the material history and characterizes its mechanical state, so that it is engraved in the material. Let us make three remarks, the first one corresponding to the BCC approach and its assumption; the second and third ones concern an experimental test of BCC theory.

R_1 : as mentioned already, i) the notion of engravement is linked to the exact distribution of contacts in the pile, and ii) the engravement is supposed to be quite sensitive to any change of contact network: if some of the contacts moves or change the closure equation is assumed to vary largely, otherwise it does not change. So, if grains are rigid, the grains shall move and the contacts shall change when, and only when, the sample deforms; hence the closure relation shall remain constant as far as the sample does not deform [3, 4].

R_2: if such a closure relation is true everywhere in the pile (with a constant value of k_2), it shall be also true after integration over a given volume [25], so that it shall be valid in mean (*i.e.* this implies $<\sigma_{XX}> =k_2 <\sigma_{ZZ}>$) and incrementally (*i.e.* $<\delta\sigma_{XX}> =k_2 <\delta\sigma_{ZZ}>= \delta<\sigma_{XX}> =k_2\ \delta<\sigma_{ZZ}>$).

R_3 : there is a well known device called the triaxial cell apparatus which is able to apply such stress fields in mean; it is sketched in Fig. (4a) for which $q = <\sigma_{XX}>-<\sigma_{ZZ}>$ and $p=<\sigma_{ZZ}>$, so that if $\sigma_{XX}=k_2\sigma_{ZZ}$ is engraved really in the pile, this shall lead to a constant value of $q/p=1/k_2-1$.

On the other hand, it is possible to run this set-up either with 2d assembly of rods [22] or with 3D granular media [23, 24]; it can be run also in different ways, the three more commonly used are very briefly summed up, but complete information can be found in [22-24] and in any text book of soil mechanics:

T_1-test: during this test, one can keep p=cste and control q in such a way that the vertical deformation $\varepsilon_{ZZ}=\varepsilon_1$ increases continuously; the result of such an experiment is sketched in Fig. (4b).

T_2-test: this test is run at constant radius ($\varepsilon_V=\varepsilon_{ZZ}$) by adjusting the ratio q/p when q is increased; this test is called oedometric test.

T_3-test: during this test, the ratio q/p is kept constant when increasing (or decreasing) q and p;

T_M-test: many other series of combinations can be used including combinations of these three kinds of sequences.

•3-a Consequences of triaxial test results on the mean closure relation

We will describe first few statements one can get from these data

Statement 1: existence of deformation: The main result of triaxial test experiments is that changing the stress distribution deforms the sample so that it generates some evolution of the contact distribution which denies in turn the possibility of constant engravement as defined byBCCW.

Statement 2: critical state does not correspond to the case of a constant closure eqation: It is also worth noting in Fig. (4b) which concerns a T_1-test on initially isotropic samples built at different densities that the ratio q/p evolves continuously from 0 till a constant ratio M is reached. Soil mechanics calls this ultimate mode of deformation when M is reached the critcal state. M does not depend on p and on the initial density. However, this constant ratio M of stress shall not be understood as an engravement in the sense of BCCW since it corresponds here to a ratio q/p which remains constant *during the deformation*; hence this ratio M is not linked to a precise contact network, but it is indeed related to a constant statistics of the contact distribution, since it has been demonstrated that this one does not evolve any more when deformation proceeds the sample staying in the critical state.

Statement 3: mean field calculation: It is then worth noticing that last part of statement 2 is in agreement with the recent mean field treatment of the stress tensor proposed by Tkachenko and Witten [18] since this one relates the stress tensor to the mean of the fabric tensor; hence, Tkachenko and Witten [18] predicts implicitly that the ratio q/p shall remain constant when the the contact distribution does not evolve, *i.e.* in the critical state. This was already derived in soil mechanics [26] under similar approximations so that both approachs are equivalent.

Statement 4: smooth evolution of q/p most of the time: It is interesting to note that q/p curves evolve slowly which implies the evolution of the closure equation to be rather smooth with deformation. This is in contradiction with the BCCW approach.

Statement 5: incremental closure relation : However, as the stress at the boundary conditions are under the operator control, any increment of stress can be performed. This implies that the incremental closure relation can be chosen at will and can evolve non smoothly.

Statement 6: Possibility of a non smooth evolution of the closure relation in the case of dense pile: one observes from Fig. (4b) that the q/p curves of dense enough piles exhibits a maximum q_M/p larger than M for a finite deformation, as deformation proceeds. So, consider the case when T_1-tests are performed by increasing continuously q at constant p and without any control of the deformation. In this case, when q reaches the maximum q_M, the system can no more evolve smoothly , it evolves abruptly and the "closure equation" q/p jumps suddenly from the value q_M/p to M; large deformation is generated in counter part.

Statement 7: Jansen modelling of silo is compatible with classical soil approach: Since Janssen theory of silo has partly motivated the approach proposed in [1], it is worth ending by discussing this approach from an experimental point of view based on triaxial test results: as a matter of fact, silo mechanics is run at constant radius if the silo walls are undeformable; so, as far as the wall friction can be neglected, the mechanical behaviour of a granular sample in a silo shall be quite similar to that one observed during T_2-oedometric test, which operates at constant radius. Indeed, this is well known in soil mechanics and has been used from long. For instance, T_2-test shows that the q/p ratio reaches a constant ratio when the load q is increased (see p. 78-79 of ref. [24]); this ratio m is different from M (defined in the T_1-test) and is equal to the value found in silo, *i.e.* the Janssen constant $1/k_2=1+m$. But increasing the loading changes the height of the sample non-reversingly; this loading imposes the evolution of the contact distribution and denies in turn the possibility of an engravement of the stress relation. Furthermore, it has been recently proposed [14] a theory based on the rheological laws of granular material to calculate the ultimate ratio q/p=m=1/k2-1 and the result fits the dependence of experimental data as a function of the friction angle. So, it turns out that the Janssen theory of silo is compatible with the classical soil mechanics approach.

So all these triaxial test results are in contradiction with the BCCW approach. However, it seems that few persons [15] try to question this approach by limiting the above results to the large stress domain so that the domain of validity of BCCW theory could be the small stress one. This point is discussed now.

•3-b Domain of validity of triaxial test:

BCCW theory is aimed at describing macroscopic piles; this means that it concerns piles larger than $1cm^3$ (when particle size is 0.3mm about). Besides, triaxial test experiment are performed with pressure p larger than 20kPa; so, in an experiment where gravity is the main stress generator, one gets this 20kPa pressure for piles larger than 1.4m (density ρ=1400kg/m^3). And one may conclude that it may exist some range (1cm-1.4m) for which triaxial test results are not valid and where BCC approach applies. Let us then discuss this point through few remarks.

Remark 1: If triaxial tests are not performed at pressure p smaller than 20kPa, it is due to the gradient of pressure generated by gravity which makes the sample response inhomogeneous and the data imprecise. This is why triaxial test experiments are planned in the microgravity program of NASA. In particular, these results may have some importance for futur landings on new planets: the LEM had so long legs because it should take off from the Moon and that scientists did not know the softness of the Moon soil.

Remark 2: however, no anomalous behavior has ever been detected when lowering p till 20kPa and one may then expect that these results can be extrapolated by continuity at smaller p too. Nevertheless, better confidence about this extrapolation would be obtained if the main mechanisms would be observed also at very low pressure. These main mechanisms are friction and dilatancy; friction is measured via the asymptotic value of the q/p ratio which is independent of strain and of p at large strain as shall be a friction coefficient; the dilatancy effect is the volume expension which is observed when increasing q; this dilatancy effect shall be observed in most piles at low pressure for any pile since the critical density above which it occurs diminishes with p . Are these facts observed at low pressure, this is what is discussed in the two next remarks.

Remark 3: Obviously, phenomena which occur near a free surface are concerned with very small pressure; this is then the case of experiments on slope stability and avalanches. It has been demonstrated recently that experimental results on slope stability and avalanches are in complete agreement with triaxial test results [27] obtained at large pressure: both experiments define the same friction angle and both are sensitive to dilatancy [28]; furthermore, in the case of avalanche, dilatancy effect has been found to increase (decrease) when gravity is decreased (increased) which is equivalent to the increase (decrease) of dilatancy effect observed at low (large) pressure in triaxial tests.

Remark 4: Indeed, Coulomb [29] has defined the maximum angle of repose of a pile as the friction angle of the granular material and he applied it to large-stress mechanics (*i.e.* to calculate stability of dams, embankments,…). Reynolds [30] has discovered the dilatancy mechanism for sand from an experiment at 0 stress, but he has generalized this effect to larger pressure too. So both these famous scientists have extrapolated the validity of these mechanisms from small stresses to large stresses, just in the reverse way as one does currently nowadays.

Endly, let us discuss i) the case of the Janssen theory of silo [12] which helped BCC to build up their theory, and ii) the case of the stress dip below a conic pile [7] which has been used by them to show how their closure equation works well. This will allow to show that the range of stress where their theory shall apply is quite large and expands also over the range of application of triaxial test results:

Remark 5: The Janssen theory of silo can be applied to calculate stress in small containers such as 1-cmdiameter tubes to real silos whose diameter can reach 10m; it can be also applied to the stress ratio in deep earth (100m or more). So, the stress range it is concerned with spans over 100 Pa (1cm earth depth) to 100 Mpa (5 km earth depth) and the stress ratio is found the same all over this range. This tends to prove that the mechanics of granular material remains similar all over this range of stress.

Remark 6: In the same way, BCCW have applied their theory to find the stress distribution in conic piles [2]; experimentally [7], the pile height ranges from few centimeters to 60cm. They have always assumed the validity of a scaling argument called RSF (*i.e.* radial stress field) scaling [31] which supposes that the mechanics of granular material remains similar all over this range of stress.

Conclusion

This paper investigates the response of granular materials to different loads in order to examine the possible existence of a relation between stresses which would be buried in the granular matter during building in order to investigate the validity of the approach proposed by BCCW [1-4] . The use of triaxial apparatus has turned out to be quite efficient, since it allows to apply to any stressed sample (σ_{xx},σ_{zz}) any increment of stress ($\delta\sigma_{xx}$,$\delta\sigma_{zz}$) in any direction.

So, it has been found using i) stress averaging and ii) triaxial-test set-up in 3d (or biaxial-test set-up in 2d) that such relations can be perturbed by any adequate change of boudary conditions. Furthermore, if the change of boundary condition is continuous, the break down of the closure equation concerns mainly the incremental (*i.e.* $\delta\underline{\sigma}$) relation whose propagation does not obey the BCCW theory hence; on the contrary, the constitutive relation between the total stress $\underline{\sigma}$ is kept constant at first order in $\delta\underline{\sigma}$. So, granular matter does not seem to behave as a fragile matter as assumed in the BCCW approach which predicts a constant constitutive law followed by sudden abrupt changes, which should exhibit large stress fluctuations.

Few other important experimental fact can be found from triaxial test data: i) when incremental stress $\delta\underline{\sigma}$ is applied, it induces the deformation of the sample in general; ii) it is worth noting, but this was not developed in the paper, that different responses are obtained when increment is positive ($\delta\sigma_{xx}$,$\delta\sigma_{zz}$) or negative (-$\delta\sigma_{xx}$,-$\delta\sigma_{zz}$). All these facts are well described by the so-called plastic theory, but are not compatible with both the BCC- & BCCW- approaches under examination.

So, as one can perform any stress increment in any direction whatever the applied stress in a whole range of stress, this denies that mechanics of granular material is governed by the burial of some stress relation. This is why the classical approach which is used in general to characterize the mechanical behaviour by some incremental relationship (ε=f(story, σ,$\delta\sigma$)) takes its meaning.

At last, on the contrary to what it was thought, it has been demonstrated that nothing could be concluded from the first 2d experiment, since this experiment was not controling or measuring stress with sufficient accuracy in enough locations; in this case, the set of differential equation obeying the closure equation and the static stress balance could have a solution compatible with the boundary conditions.

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On the other hand, H.G.B. Allersma [http://dutcgeo.ct.tudelft.nl/allersma/hgball.htm] has investigated the penetration of a cone in a photoelastic granular material ; the photography of his results is not in agreement with the above BCC theory since stress propagates into the material occupying a cone volume whose axis is vertical.
[14] For instance, in order to illustrate the important role plaid by strain in the silo case, this previous version was demonstrating the closure relation (ʃxx=k ʃzz) in the case of silo and was deriving the precise value of k from the quasi-static rheology of granular materials (*i.e.* from quasi-static stressstrain relations) ; this point is published now in P. Evesque, *J. de Physique I France* **7** (1997), 1501-12.
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[17] M.E. Cates, "Actually, Evesque's ideas ... have already strongly influenced our more recent work", (private communication)(referee report) .
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[20] x and z stand for the horizontal and vertical axes respectively, 〉 for the pile density, g for gravity and co2=k2.= ʃxx/ ʃzz . Other stress components can be deduced from the closure relation
[21] As photoelasticity technique requires the deformation of grains, the result observed by Pouliquen requires a small compression of the two oblique lines of grains supporting the extra load; this can be obtained in a perfect hexagonal lattice if , and only if, the contact between the grains pertaining to these two lines with the grains of the four adjacent lines evolve slightly. As Pouliquen's

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experiment is a real experiment, the coefficient of friction between the grains is non zero. So, this sliding can occur only if the extra force transmitted via the two oblique lines overpasses the friction force which can be mobilized between these two lines and the four adjacent oblique lines. Near the pile bottom, the friction force is proportional to the pile height h, since it is proportional to half the weight of a grain column due to the natural action of gravity. So,

integrating all the friction forces along the oblique path leads to a total friction scaling as h_2 and the experimental result obtained by Pouliquen is observed most likely if and only if the applied extra force is large. Furthermore, this shows that one cannot neglect the evolution of the contact distribution; for instance, its finding will be the same starting from a slightly perturbed hexagonal configuration.

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Figure caption:

Figure 1: 2D experimental set-up:

Fig. 1a: Stress propagation "according to ref [1]".

Fig. 1b: the structure on which the granular medium lays is made of two parallel U-shaped structure with a void in between them

Fig. 1c: rods are laid on the structure; they are 5mm diameter and 6cm long; probe rods P are 3cm long and 5mm diameter. This probe is carried by a structure which passes in the space between the two U and which lays on the scales S2. The set-up lays on two other scales S1 and S3. Some additionnal weight F may be applied at different points of the top layer; this induces a change of the weight measured by each scales. Two of these scales are hard, the third one is much softer.

Figure 2: A rectangular pile is carried in three points by three scales (see Fig. 1b). The variation of response of each scales S1-S1o,(squares), S2-S2o,(triangles), S3-S3o (losanges) when an overload (M=200g or 50g), is plotted as a function of the overload position on the top of the rectangular pile. The probe is linked to S2 and is made either of 1 or 5 rods, (see caption title). The variation of the weight does not depend on the initial values of S1o, S2o, S3o, nor on the

probe size, nor on the pile height (21 or 27 rods layers), but on the position of the less rigid scales.

Fig. 2a: S2 is the softest-spring balance; this fixes the response of S2 to be constant.

Fig. 2b: S3 is the softest-spring balance; this fixes the response of S_3 to be constant.

Figure 3: how to get a stress which looks like propagating "along two lines" using special boundary conditions and elastic medium: a soft balance applies two equal forces f at two different locations of the bottom surface of a material; the top surface of the material also is loaded by a small mass m at its center (2f>mg). If the mass m is moved away from the top surface and is placed on the the soft balance, each force f applied to the medium decreases of a quantity ™f=mg/2 so that unload mg "seems to have propagated" (in straight line) from the top to the

bottom. Replacing m on the top surface just in the center forces the applied force mg to propagate from the top to the bottom and the response is localized at the contact points with the soft scales. However, the inclination of the "straight line" may be varied at will either by changing the position of the forces f since it is linked to the chosen boundary condition or by changing the location where m is placed.

Figure 4: A 3-D granular medium made of rigid grains can deform under stress.

Fig. 4a: a typical axisymmetic triaxial test set-up consists of a plastic cylindrical bag which contains the granular medium; it is immersed in a container filled with water at pressure p= ʃ$_{xx}$= ʃ3 and maintained in between two vertical pistons which applies a variable vertical overload q= ʃ$_{zz}$ - ʃ$_{xx}$= ʃ1- ʃ3 .

Fig. 4b: Typical results obtained with a triaxial cell, when ʃ3 remains constant. The mechanical behaviour is summed up by the knowledge of the three following parameters ʃ3 , the deviatoric stress q=(ʃ1- ʃ3) which characterizes the shearing force, the vertical strain ∑1 and the volumetric strain ∑v=∑$_{1}$+2∑$_{3}$ =∑$_{zz}$-∑$_{xx}$ or the specific volume v . Typical experimental result obtained with the same sand packed initially isotropically either at at two different densities (_____ dense ; ____ _ _____ loose) . When the pile is dense, one observes the dilatancy mechanism which is associated with a bump on the q vs. ∑1 curve. One remarks also the q/p asymptotic behaviour . It is a measure of the friction angle.

One obtains that the asymptotic value v_c depends on p, but not on the initial specific volume vo and that the asymptotic value M of q/p does not depend on p and vo. The transient behaviour depends on both v_o and p.

Rapport d'Activité de P. Evesque, CNRS 2009-2010 p. 68/